PHYSICS 231 | Final Examination | Name: Jake Bobowski
2012/13 Term 1 | | Student Number: Sol'ns

Final Exam (80 points)
Free Response: Write out complete answers to the following questions. Show your work.

(5pts) **1.** Use Euler's equation ($e^{\pm j\phi} = \cos\phi \pm j\sin\phi$) to prove the following trig identities:

$$\cos(\phi_1 \pm \phi_2) = \cos\phi_1\cos\phi_2 \mp \sin\phi_1\sin\phi_2$$
$$\sin(\phi_1 \pm \phi_2) = \sin\phi_1\cos\phi_2 \pm \cos\phi_1\sin\phi_2$$

$$e^{j(\phi_1 \pm \phi_2)} = e^{j\phi_1} e^{\pm j\phi_2}$$

$$\cos(\phi_1 \pm \phi_2) + j\sin(\phi_1 \pm \phi_2) = (\cos\phi_1 + j\sin\phi_1)(\cos\phi_2 \pm j\sin\phi_2)$$

$$= (\cos\phi_1\cos\phi_2 \mp \sin\phi_1\sin\phi_2)$$

$$+ j(\sin\phi_1\cos\phi_2 \pm \cos\phi_1\sin\phi_2)$$

$$\therefore \cos(\phi_1 \pm \phi_2) = \cos\phi_1\cos\phi_2 \mp \sin\phi_1\sin\phi_2$$

$$\sin(\phi_1 \pm \phi_2) = \sin\phi_1\cos\phi_2 \pm \cos\phi_1\sin\phi_2$$

5 pts

(10pts) **2.** The quality factor Q of an LRC-circuit is defined to be the ratio of the resonance frequency to the width of the resonance. For the parallel LRC-circuit, the Q-factor is given by:

$$Q = R\sqrt{\frac{C}{L}}$$

(a) If $R \pm \delta R$, $C \pm \delta C$, and $L \pm \delta L$ are determined by experimental measurements, what is the uncertainty in the calculated value of Q? Find an expression in terms of R, δR, C, δC, L, and δL.

(b) If R and C are both known within to 5% and L is known to within 10%, what is the percent uncertainty in Q?

(a)
$$\delta Q^2 = \left(\frac{\partial Q}{\partial R}\delta R\right)^2 + \left(\frac{\partial Q}{\partial C}\delta C\right)^2 + \left(\frac{\partial Q}{\partial L}\delta L\right)^2$$

$$= \left(\sqrt{\frac{C}{L}}\,\delta R\right)^2 + \left(\frac{R}{2}\frac{1}{\sqrt{LC}}\,\delta C\right)^2 + \left(\frac{R}{2}\sqrt{\frac{C}{L}}\,\frac{\delta L}{L}\right)^2$$

(b)
$$\delta Q^2 = \left(R\sqrt{\frac{C}{L}}\,\frac{\delta R}{R}\right)^2 + \left(\frac{1}{2}R\sqrt{\frac{C}{L}}\,\frac{\delta C}{C}\right)^2 + \left(\frac{1}{2}R\sqrt{\frac{C}{L}}\,\frac{\delta L}{L}\right)^2$$

$$= \left(Q\frac{\delta R}{R}\right)^2 + \left(\frac{1}{2}Q\frac{\delta C}{C}\right)^2 + \left(\frac{1}{2}Q\frac{\delta L}{L}\right)^2$$

$$\therefore \left(\frac{\delta Q}{Q}\right)^2 = \left(\frac{\delta R}{R}\right)^2 + \left(\frac{1}{2}\frac{\delta C}{C}\right)^2 + \left(\frac{1}{2}\frac{\delta L}{L}\right)^2$$

$$= (0.05)^2 + \left(\frac{1}{2}0.05\right)^2 + \left(\frac{1}{2}0.10\right)^2$$

$$\boxed{\therefore \frac{\delta Q}{Q} = 0.075 \Rightarrow 7.5\%}$$

10 pts

(10pts) **3.** (a) Staring from $V_C = q/C$, and assuming time-harmonic (sinusoidal) signals, derive an expression for the impedance Z_C of a capacitor.

(b) Staring from $V_L = L\frac{di}{dt}$, and assuming time-harmonic (sinusoidal) signals, derive an expression for the impedance Z_L of an inductor.

(a) $V_c = \frac{q}{C} = i Z_c$ assume $i = I_0 e^{j(\omega t + \phi)}$

$q = \int i\,dt = \frac{1}{j\omega} I_0 e^{j(\omega t + \phi)} = \frac{i}{j\omega}$

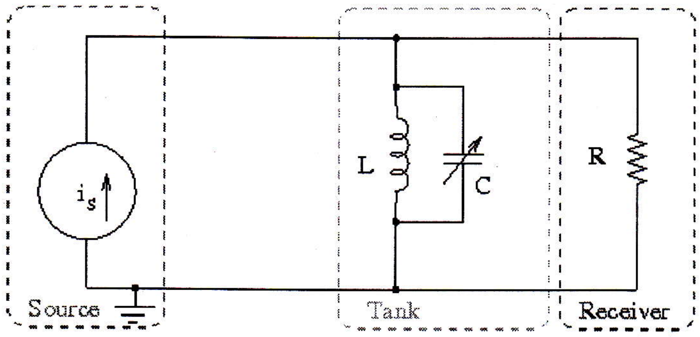
$\therefore V_c = \frac{i}{j\omega C} = i Z_c \quad \therefore \boxed{Z_c = \frac{1}{j\omega C}}$

(b) $V_L = L\frac{di}{dt} = i Z_L \qquad \frac{di}{dt} = j\omega I_0 e^{j(\omega t + \phi)} = j\omega i$

$\therefore V_L = L(j\omega i) = i Z_L \quad \therefore \boxed{Z_L = j\omega L}$

10 pts

(15pts) **4.** If we want to select signals within a range of frequencies from an input source which contains a wide range of frequencies, a resonant tank circuit can be used. In this problem, i_S is a current source.



(a) Show that when $\omega \approx 1/\sqrt{LC}$ the impedance of the parallel LC combination is very large, such that the signal is passed to the receiver (resistor R). On the hand, show that when ω is far from $1/\sqrt{LC}$, the impedance of the tank circuit is very low such that signal is not passed to the receiver.

$$Z = Z_L \parallel Z_C = \frac{Z_C Z_L}{Z_C + Z_L} = \frac{\frac{1}{j\omega C} j\omega L}{\frac{1}{j\omega C} + j\omega L} = \frac{L/C}{j\left(\omega L - \frac{1}{\omega C}\right)}$$

If $\omega = \frac{1}{\sqrt{LC}}$ then $\omega L - \frac{1}{\omega C} = \sqrt{\frac{L}{C}} - \sqrt{\frac{L}{C}} = 0$

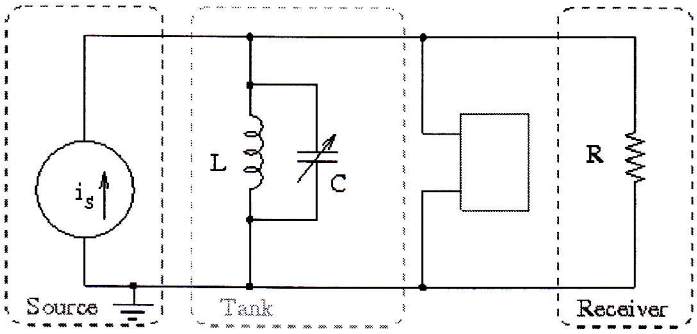
$\therefore Z \to \infty$. $\therefore$ current goes through R rather than LC combo.

If $\omega \to 0$ or ∞ $\quad \omega L - \frac{1}{\omega C} \to \pm\infty \quad \therefore Z \to 0$

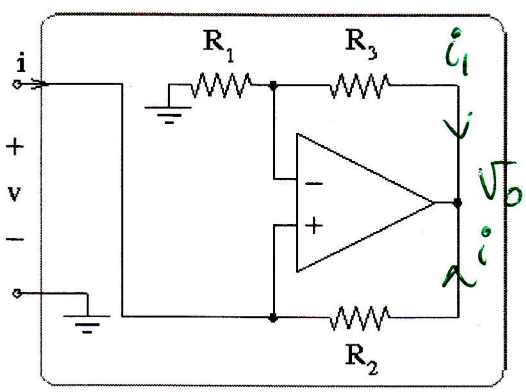
$\therefore$ LC combo shorts R, no current (v. little) through R.

15 pts

(b) Recall from problem 2 that the width of the resonance, or equivalently, the range of frequencies selected by the tank circuit is determined by the Q of the parallel LRC circuit. To increase the selectivity of the tank circuit, we want to increase the Q by using a sub-circuit between the tank and receiver to increase the effective resistance in parallel with the LC combination.



The proposed sub-circuit is shown below. Determine the equivalent resistance R_{eq} of this sub-circuit. That is, find the ratio $v/i \equiv R_{eq}$ where v is the labelled voltage across the input terminals and i is as shown in the figure. *Hint: You should find that R_{eq} is negative!*



Op amp Golden rules

--- no current into op amp inputs.

--- $V_+ = V_-$ ---> $\boxed{V_+ = V_- = v}$

$$0 - i_1 R_1 = v_- = v \Rightarrow i_1 = -\frac{v}{R_1}$$

$$v - iR_2 + i_1(R_1 + R_3) = 0$$

$$v\left[1 - \frac{R_1 + R_3}{R_1}\right] = iR_2 \quad \therefore \quad v\left[-\frac{R_3}{R_1}\right] = iR_2$$

$$\therefore \frac{v}{i} = \boxed{R_{eq} = -\frac{R_1 R_2}{R_3}}$$

0 pts

(c) Finally, suppose $R = 2000\ \Omega$. What is the required value of R_{eq} to make $R||R_{eq} = 10\ \mathrm{k\Omega}$? If $R_1 = 2R_3$, what is the required value of R_2?

$$R // R_{eq} = \frac{R R_{eq}}{R + R_{eq}} = 10\ k\Omega$$

$R = 2k\Omega$

$$\therefore \frac{2 R_{eq}}{2 + R_{eq}} = 10 \Rightarrow 2R_{eq} = 20 + 10R_{eq}$$

$$\therefore 20 = -8R_{eq}$$

$$\therefore \boxed{R_{eq} = -2.5\ k\Omega}$$

$$-2.5\ k\Omega = -\frac{R_1 R_2}{R_3} = -\frac{2\cancel{R_3}\, R_2}{\cancel{R_3}}$$

$$\therefore 2.5\ k\Omega = 2R_2$$

$$\therefore \boxed{R_2 = 1.25\ k\Omega}$$

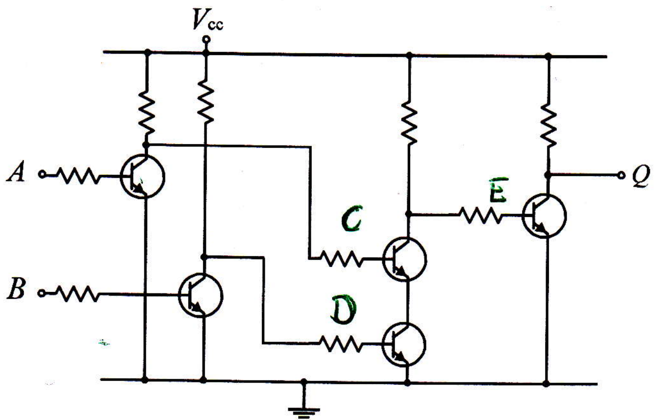
0 pts

(10pts) **5.** (a) Consider the following five digital operations:

$$Q = A \cdot B,\ \overline{Q} = \overline{\overline{A} \cdot \overline{B}},\ Q = \overline{A \cdot B},\ Q = \overline{A} + \overline{B},\ Q = \overline{A} \cdot \overline{B}$$

How many unique operations are represented in the list?

(b) Identify which of the operations listed in (a) are performed by the transistor circuit shown below? In the circuit below, lines that intersect without a dot, are **not** connected.



(a) $Q = A \cdot B$ (i)

$\overline{Q} = \overline{\overline{A} \cdot \overline{B}} \Rightarrow Q = \overline{A} \cdot \overline{B}$ (ii)

$Q = \overline{A \cdot B} = \overline{A} + \overline{B}$ (iii)

$Q = \overline{A} + \overline{B}$ (iv)

$Q = \overline{A} \cdot \overline{B}$ (v)

3 unique operations.

b)

A	B	C	D	E	Q
0	0	1	1	0	1
0	1	1	0	1	0
1	0	0	1	1	0
1	1	0	0	1	0

transistor circuit perfoms

$Q = \overline{A} \cdot \overline{B} \Rightarrow \overline{Q} = \overline{\overline{A} \cdot \overline{B}}$

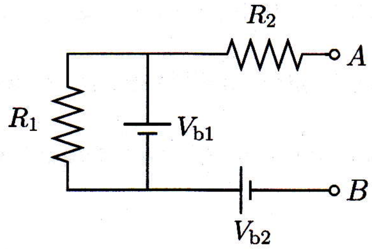
check

A	B	$\overline{A}$	$\overline{B}$	$\overline{A} \cdot \overline{B}$
0	0	1	1	1
0	1	1	0	0
1	0	0	1	0
1	1	0	0	0

✓

10 pts

(10pts) **6.** Calculate the Thevenin equivalent circuit parameters. Draw the Thevenin equivalent circuit.



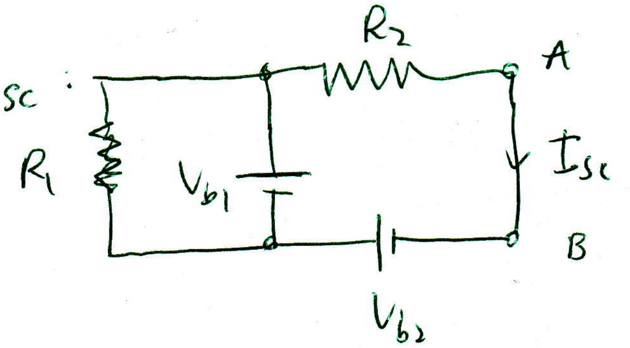
$V_T = V_{oc}$

no current. 0

$V_B + V_{b2} + V_{b1} + \cancel{\Delta V_{R_2}} = V_A$

$\therefore V_{oc} = V_A - V_B = \boxed{V_{b1} + V_{b2} = V_T}$

I_{sc} :

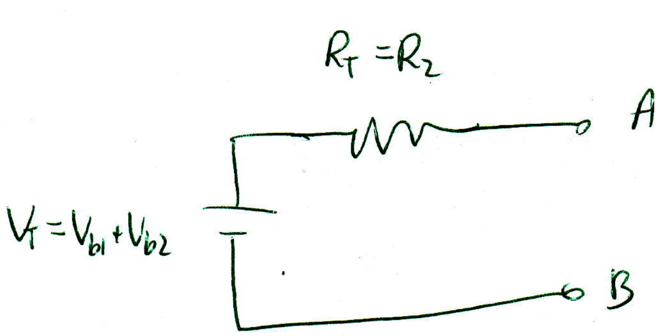


$V_B + V_{b2} + V_{b1} - I_{sc} R_2 = V_B$

$\therefore I_{sc} = \dfrac{V_{b1} + V_{b2}}{R_2} = \dfrac{V_T}{R_2}$

$$\boxed{R_T = \frac{V_T}{I_{sc}} = R_2}$$

Equil. circuit.



10 pts

 Name: Sol'ns

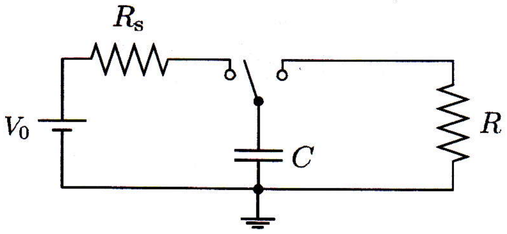
(10pts) **7.** In this circuit $V_0 = 410$ V, $R_S = 1200\ \Omega$, $C = 270\ \mu$F, and $R = 10\ \Omega$. Assume that the switch has been positioned to the right (connecting C and R) for a very long time.

(a) At $t = 0$, the switch is flipped to the left. What is the voltage on the capacitor at $t = 0.9$ s?

(b) At $t = 0.9$ s, the switch is flipped back to the right. Immediately after the flip, what is the current through R?

$C = \frac{q}{V_c}$

(c) What is the voltage across R at time $t = 0.91$ s?



(a) Initially, $V_c(t=0) = 0 \therefore q_0 = 0$

when switch ~~is~~ flipped to left start charging cap

$$V_c = V_0\left(1 - e^{-t/R_sC}\right) \qquad V_0 - iR - V_c = 0$$

$$\therefore V_c(t = 0.9\text{s}) = \boxed{384.5\text{ V}}$$

(b) when flipped to right, $V_c + iR = 0$

$$\therefore i(0.9\text{s}) = \frac{V_c(0.9\text{s})}{R} = \boxed{38.45\text{ A}}$$

(c) Discharging:

$$V_c = V_c(0.9\text{s})\, e^{-t'/RC} \qquad (t' = t - 0.9\text{s})$$

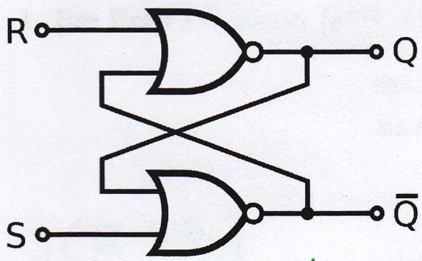
$$= V_c(0.9\text{s})\, e^{-0.01/RC}$$

$$= \boxed{9.47\text{ V}}$$

10 pts

Sol'ns

(10[pts]) **8.** In lectures and labs we studied the R-S flip-flop built from NAND gates. Consider the flip-flop circuit below constructed from NOR gates. Describe the operation of this circuit. In particular, what are the states of Q and $\overline{Q}$ when R was last HI? What about if S was last HI? You justify your answers to receive full credit. In the circuit below, lines that intersect without a dot, are **not** connected.



NOR

0	0	1
0	1	0
1	0	0
1	1	0

Assume initial states: $Q=0$ $\overline{Q}=1$ & $R=S=0$

R	S	Q	$\overline{Q}$
0	0	0	1 ✓
0	0	1	0 ✓
1	1	0	1 ×

is this consistent?

$\boxed{(R, S) \to (0\ 0) \text{ stable state}}$

Examine what happens when one of R or S goes HI then returns to LO

sequence

R	S	Q	$\overline{Q}$
0	0	0	1
1	0	0	1
0	0	0	1
0	1	1	0
0	0	1	0
1	0	0	1
0	0	0	1

If R last HI; $Q=0$, $\overline{Q}=1$

If S last HI; $Q=1$, $\overline{Q}=0$

10 pts